Discrete-Event Simulation: Examples

Lawrence M. Leemis and Stephen K. Park, Discrete-Event Simulation - A First Course, Prentice Hall, 2006

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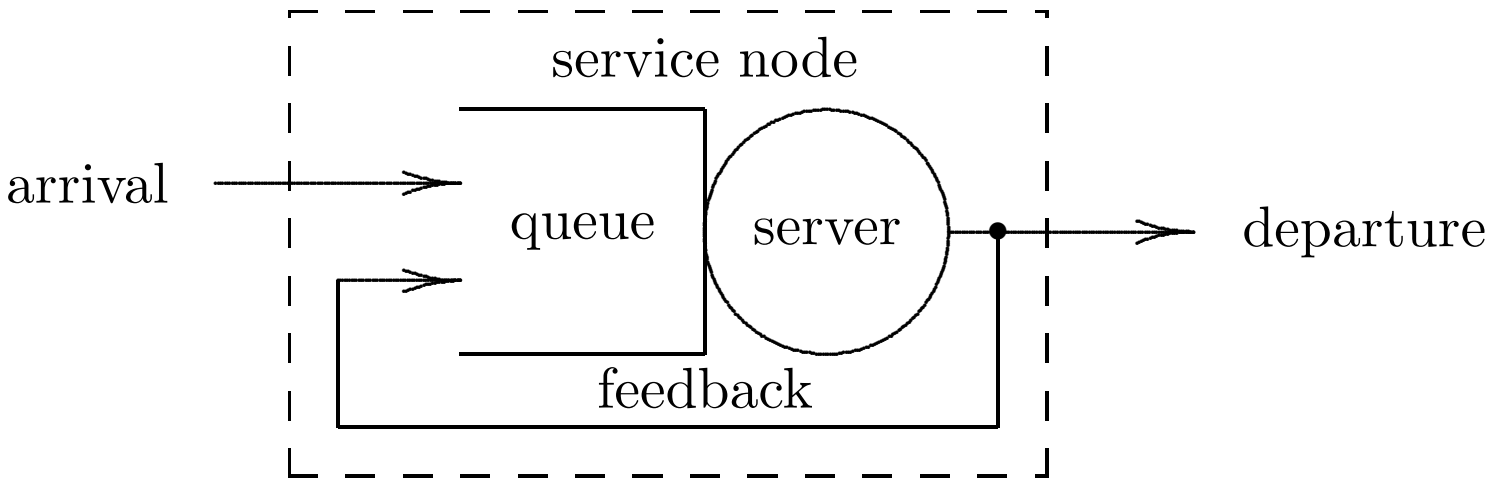
SSQ with Immediate Feedback

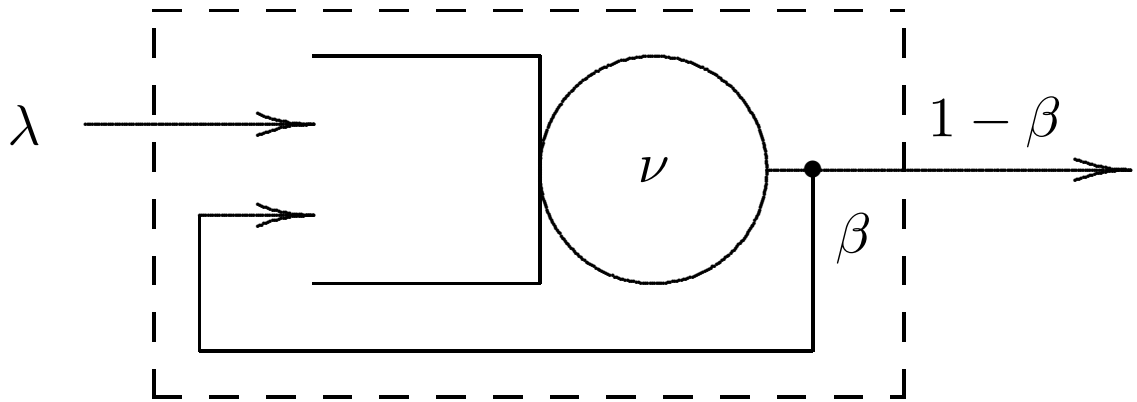
We now consider *immediate feedback* in the single-server service node model, i.e.,

- the possibility that the service a job just received was incomplete or otherwise unsatisfactory and the job feedback immediately to request service once again.
- Completion of service and departure now have different meanings
  - At the completion of service, jobs either depart the service node forever or immediately feed back and once again seek service.



Model Considerations

- As it completes its service, each job departs the service node with probability $1 - \beta$, or feeds back with probability β
  - When feedback occurs the job joins the queue consistent with the queue discipline
  - The decision to depart or feed back is random with feedback probability β



  - λ is the arrival rate
  - ν is the service rate

Model Considerations

- Feedback is independent of past history
- In theory, a job may feed back arbitrarily many times
- Typically, β is close to 0.0

GetFeedback Method

```
int GetFeedback(double beta) /* 0.0 <= beta < 1.0 */
{
    SelectStream(2);
    if (Random() < beta)
    return (1); /* feedback occurs */
    else
    return (0); /* no feedback */
}
```

Statistical Considertations

- Index $i = 1, 2, 3, \ldots$ counts jobs that enter the service node
  - fed-back jobs are not recounted
- Using this indexing, all job-averaged statistics remain valid
- We must update delay times, wait times, and service times for each feed back
- Jobs from outside the system are merged with jobs from the feedback process by the positive additive factor $\lambda \overline{x} \nu$
- Note that $\overline{s}$ increases with feedback but $1/\nu$ is the average service time per request

Example 3.3.1

job index	1	2	3	4	5	·	6	·	7	8	·	9	...
arrival/ feedback	1	3	4	7	10	**13**	14	**15**	19	24	**26**	30	...
service	9	3	2	4	7	5	6	3	4	6	3	7	...
completion	10	**13**	**15**	19	**26**	31	37	**40**	44	50	53	**60**	...

- At the computational level, some algorithm and data structure is necessary

Algorithm and Data Structure Considerations

- Need to insert fed-back jobs into arrival stream
- Need a data structure to hold fed-back jobs and arrivals

In-Class Exercise L7-1 (Hints in Next Slide)

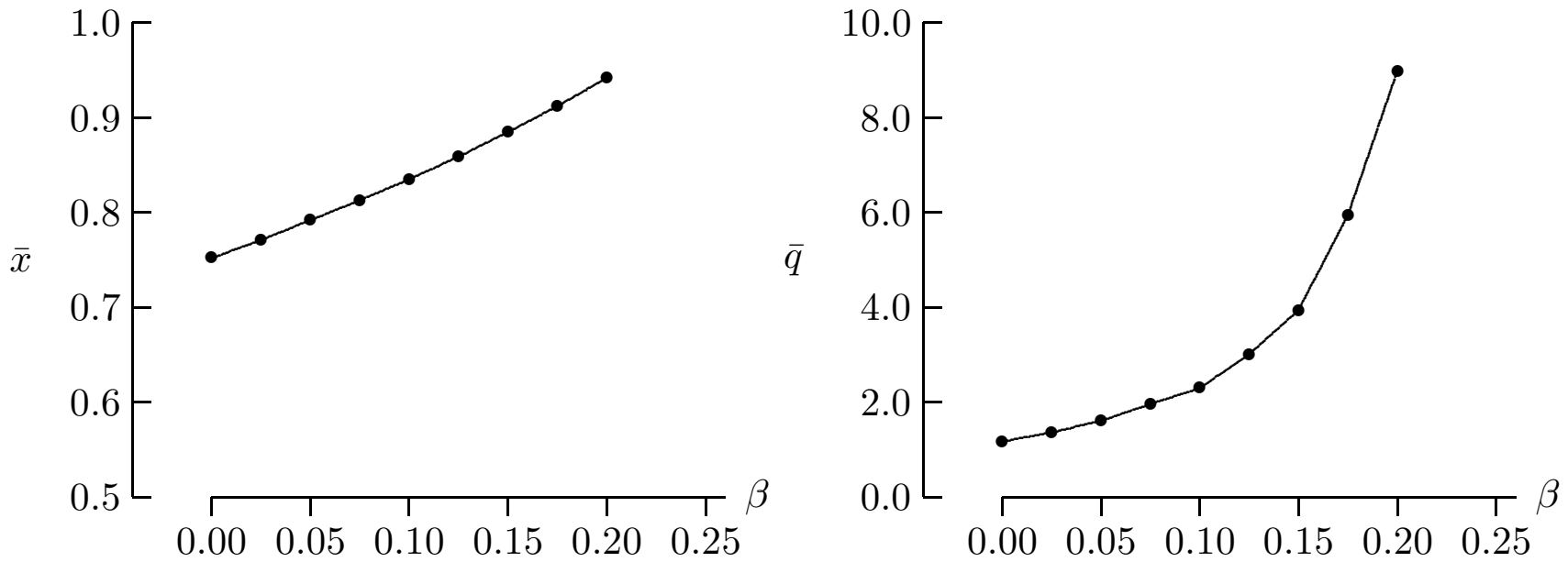
- Make a copy of *ssq2* and name the copy as *ssq2ex332*.
- Modify *ssq2ex332* to account for immediate feedback
  - Use an array to store arrivals including both fed-back and new arrivals
  - hint: how big the array shoud be? which array element contains earliest arrival in the array including the fed-back one?
- Use the following parameters
  - The arrival process has *Exponential*(2.0) random-variate interarrival times
  - The service process has *Uniform*$(1.0, 2.0)$ random-variate service times
  - The feed-back probability is $0.0 \leq \beta < 1.0$. To illustrate the effect of feedback, the modified program is to simulate the operation of a single-server service node with 9 different values of levels of feedback, varied from $\beta = 0.0$ to $\beta = 0.20$
  - In each case, $100,0000$ arrivals are to be simulated.
- Graph utilization $\overline{x}$ as a function of β and the average number in the queue $\overline{q}$ as a function of β
- Submit the programs, the graphs, the simulation results in Blackboard

In-Class Exercise L7-1: Hints

- Change the main method/function to *SimulateOnce(double feedbackProbability)*
  - Let it print out the output as *CSV* format;
  - Declare an array to hold arrivals including fed-back arrivals
  - Insert arrivals to the array
  - Search earliest arrival from the array
- Add *int GetFeedback(double beta)* method/function in slide 5 to the program
- Add a main method/function that *has* the logic similar to the following,

```
double step = 0.2/8.0, feedbackProbability;   int i;
for (i = 0; i <= 8; i ++) {
    feedbackProbability = step * i;
    SimulateOnce(feedbackProbability);
}
```

Example 3.3.2: Sample Result from In-Class Exercise L7-1



- Left-hand side: Utilization $\overline{x}$ versus feedback probability β
- Right-hand side: The average number of jobs in the queue $\overline{q}$ versus feedback probability β

Discussion

- Is *array* a good choice to store arrivals in In-Class Exercise L7-1?
- What checks can you do to have an improved confidence on the program and the result?

Flow Balance and Saturation

- Jobs flow into the service node at the average rate of λ
- To remain flow balanced jobs must flow out of the service node at the same average rate
- The average rate at which jobs flow out of the service node is

$$\overline{x}(1-\beta)\nu$$

- Flow balance is achieved when $\lambda = \overline{x}(1-\beta)\nu$
- Saturation occurs when $\overline{x} = 1$ or as $\beta \to 1 - \lambda/\nu = 0.25$

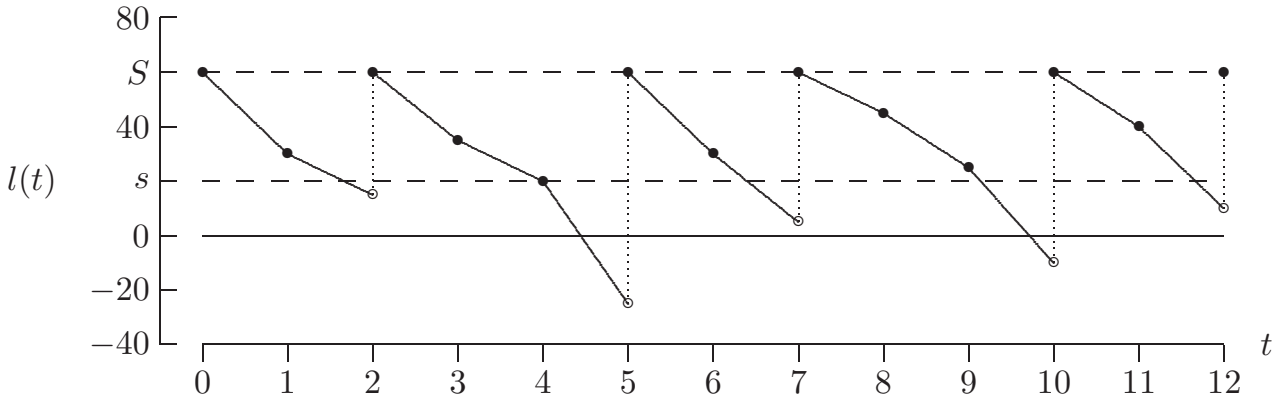
Simple Inventory System with Delivery Log

An extension to the periodic-review simple-inventory-system model

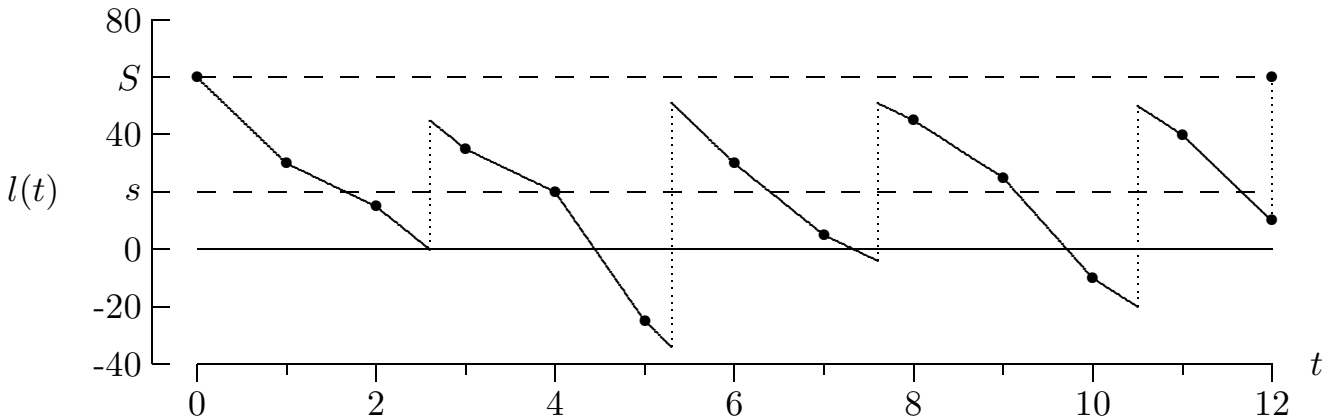
- *Delivery lag* (or *lead time*)
  - An inventory replacement order placed with the supplier will not be delivered immediately
  - There will be a *lag* between the time an order is placed and the time the order is delivered
  - Lag is *assumed* to be random and independent of order size

Inventory Levels With and Without Delivery Lag

- Without lag, inventory jumps occur only at inventory review times

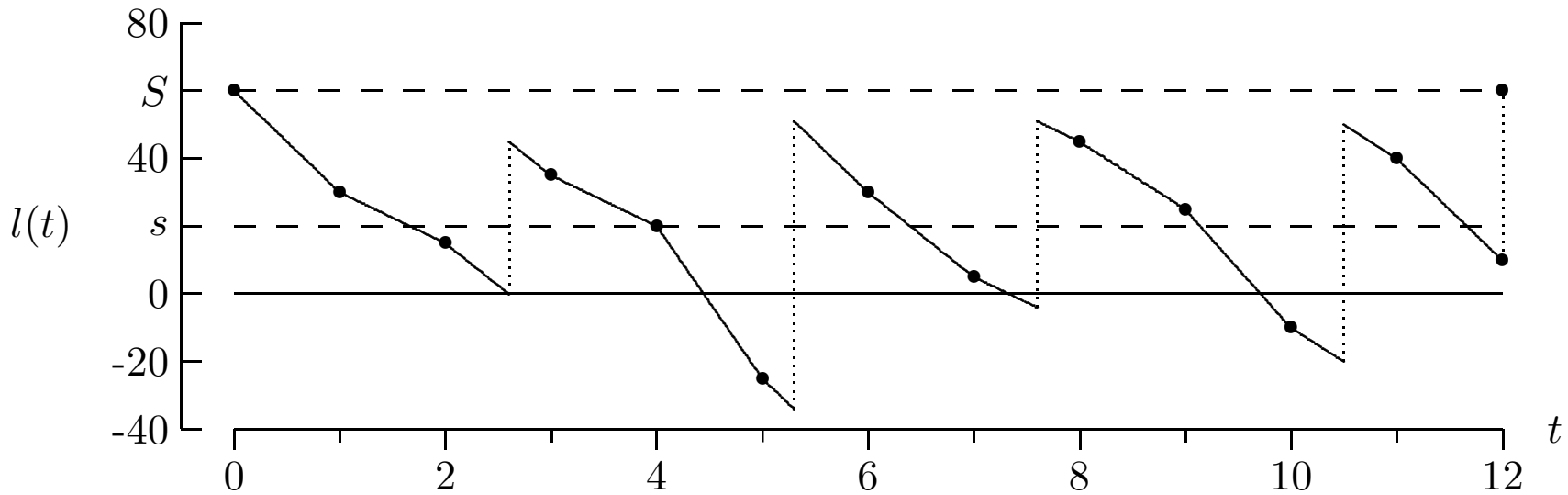


- With delivery lag, inventory jumps occur at arbitrary times



SIS with Lag

- The last order is *assumed* to have no lag
- We *assume* that orders are delivered before the next inventory review
- With these assumptions, there is no change to the specification model. However, there is a significant change in how the system statistics are computed.



SIS with Lag: Statistical Consideration

There is a significant change in how the system statistics are computed.

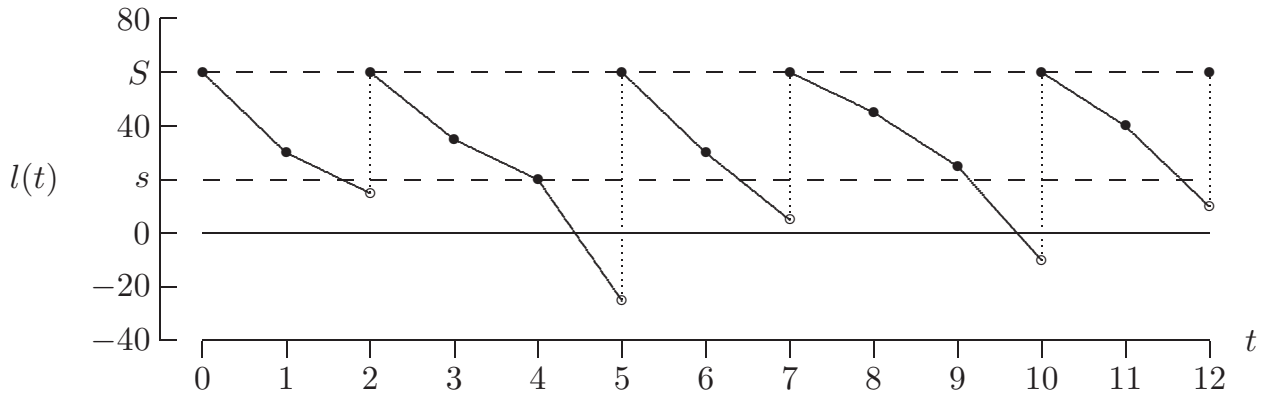


Figure : Inventory level *without* delivery lag

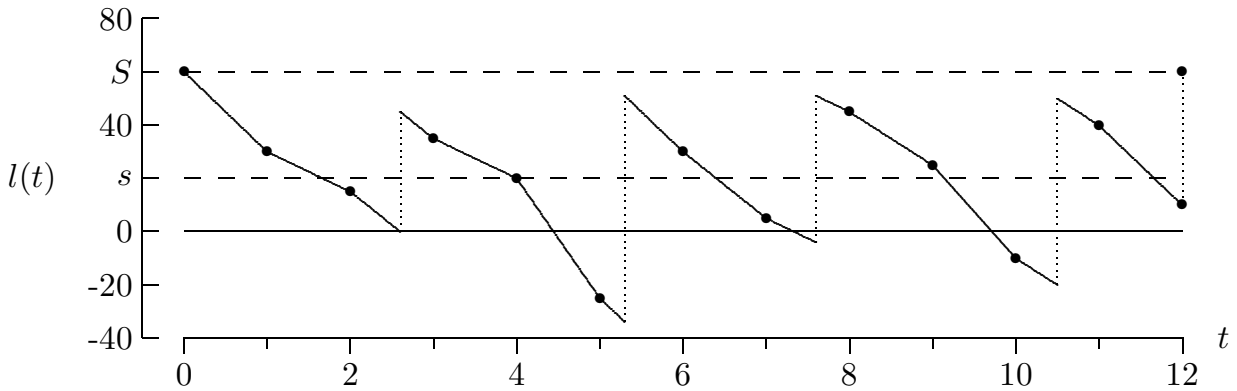


Figure : Inventory level *with* delivery lag

Time Evolution of Inventory Level

How should we revise *Algorithm 1.3.1*?

Algorithm 1.3.1

$l_0 = S$;
$i = 0$;
while (more demand to process) {
  $i++$;
  if $(l_{i-1} < s)$
    $o_{i-1} = S - l_{i-1}$;
  else
    $o_{i-1} = 0$;
  $d_i = GetDemand()$;
  $l_i = l_{i-1} + o_{i-1} - d_i$;
}
$n = i$;
$o_n = S - l_n$
$l_n = S$;
return l_1, l_2, l_3, ...l_n and o_1, o_2, ...o_n;

Statistical Considerations

- If $l_{i-1} \geq s$ the equations for $\bar{l}_i^+$ and $\bar{l}_i^-$ remain correct since there is no order, hence no delivery lag
- When delivery lag occurs the time-averaged holding and shortage intervals must be modified
  - The delivery lag for interval i is $0 < \delta_i < 1$

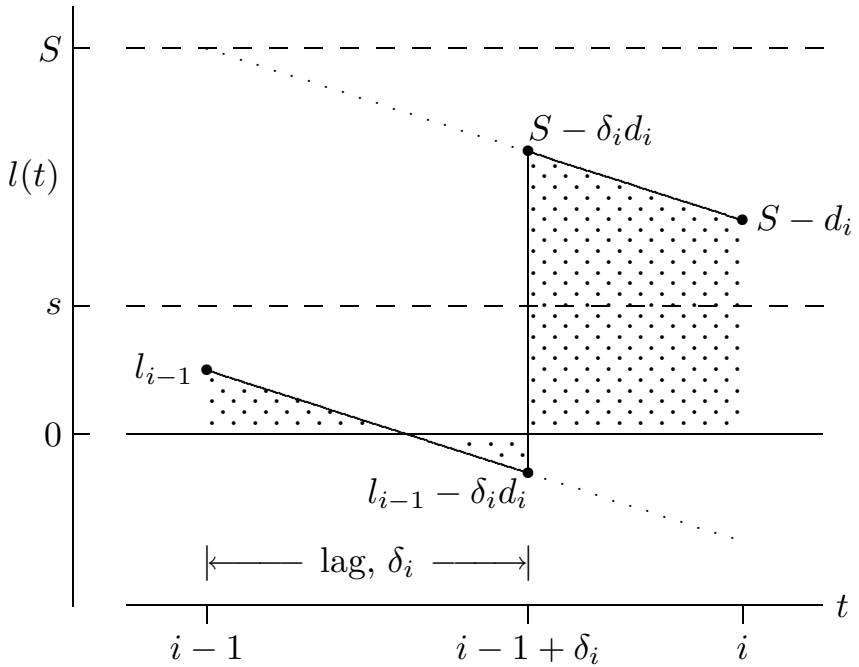


Figure : Inventory level *with* delivery lag

Time Evolution of Inventory Level

How should we revise *Algorithm 1.3.1*?

Algorithm 1.3.1

$l_0 = S$;
$i = 0$;
while (more demand to process) {
  $i++$;
  if ($l_{i-1} < s$)
    $o_{i-1} = S - l_{i-1}$; $\delta_i = GetDeliveryLag()$
  else
    $o_{i-1} = 0$;
  $d_i = GetDemand()$;
  /* compute statistics before the delivery arrives */
  .....
  /* compute statistics after the delivery arrives */
  .....
  $l_i = l_{i-1} + o_{i-1} - d_i$;
}
$n = i$;
$o_n = S - l_n$
$l_n = S$;
return l_1, l_2, l_3, ...l_n; o_1, o_2, ...o_n; and δ_1, δ_2, ...δ_n;

Revising Algorithm 1.3.1

- Determining delivery lag
  - Recall: lag is *assumed* to be random and independent of order size
- Computing statistics
  - Calculations of statistics remain the same: average order, average demand, and relative frequency of setups
  - Calculations of statistics need to change: time-averaged holding level, and time-averaged shortage level
    - How to calculate: view the calculation as estimating the area of trapezoids.

Consistency Checks

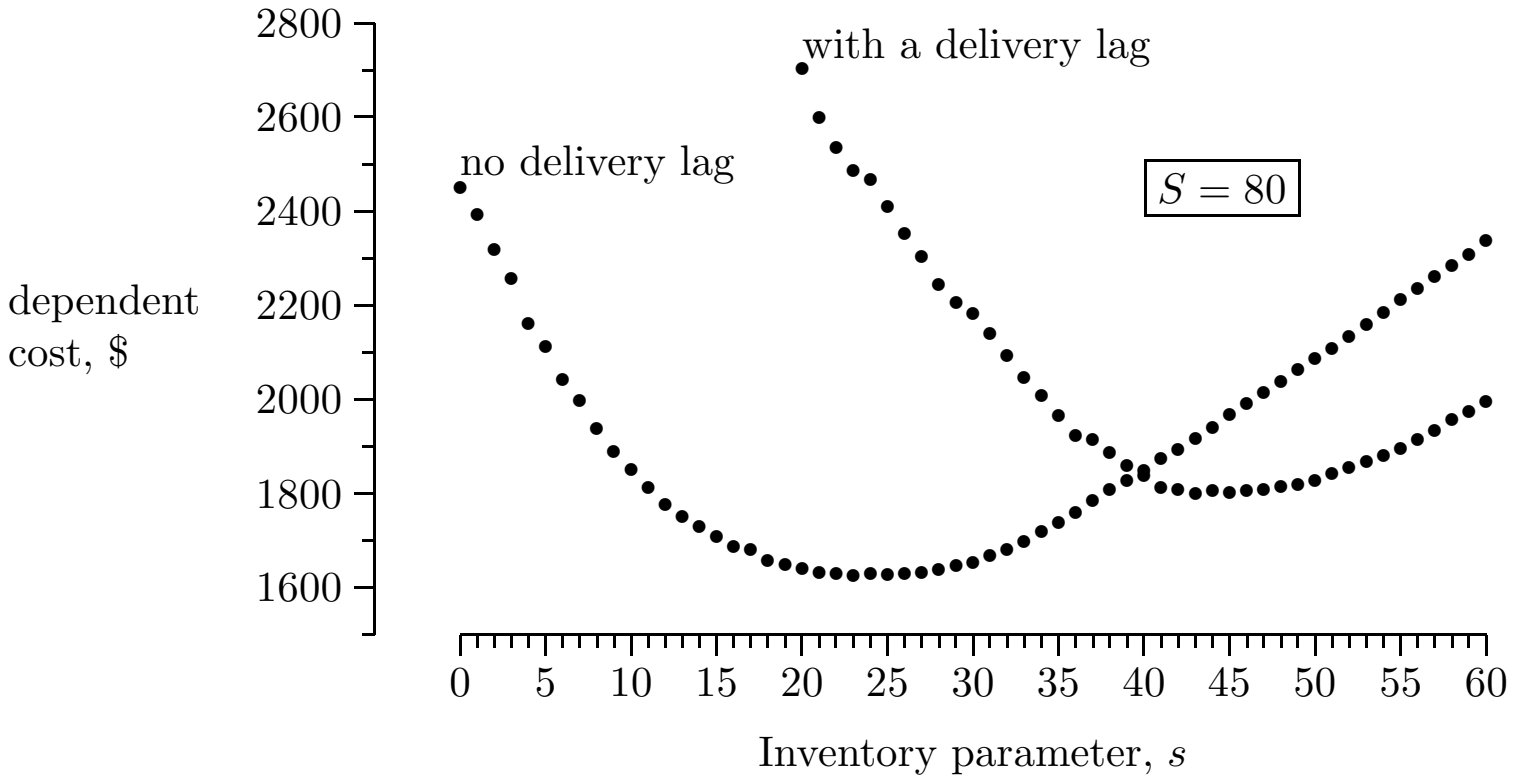
- It is fundamentally important to verify extended models with the parent model
  - Set system parameters to special values
- Set $\beta = 0$ for the SSQ with feedback
  - Verify that all statistics agree with parent
- Using the library rngs facilitates this kind of comparison
- It is a good practice to check for intuitive "small-perturbation" consistency
  - Use a small, but non-zero β and check that appropriate statistics are slightly larger

Example 3.3.3

- For the SIS with delivery lag, $\delta_i = 0.0$ if and only if no order during i-th interval, $0 < \delta_i < 1.0$;
- Otherwise, the SIS is lag-free if and only if $\delta_i = 0.0$ for all i
- If (S, s) are fixed then, even with small delivery lags:
  - $\overline{o}$, $\overline{d}$, and $\overline{u}$ are the same regardless of delivery lag
  - Compared to the lag-free system, $\overline{I}^+$ will decrease
  - Compared to the lag-free system, $\overline{I}$ will increase or remain unchanged

Example 3.3.4

- Delivery lags are independent Uniform(0.0, 1.0) random variates



- Delivery lag causes $\overline{I}^+$ to decrease and $\overline{I}$ to increase or remain the same
- $C_{Setup} = \$1,000$, $C_{hold} = \$25$ and $C_{short} = \$700$ cause it to shift up and to the left

In-Class Exercise L7-2

You will complete examples 3.3.3 and example 3.3.4, for which, you will revise *sis2* program.

- Make a copy of the *sis2* program. Name the copy as *sis2ex72* that you will be working on.
- Replace library *rng* by library *rngs*
- Use two random streams to generate demand and delivery lag
  - Revise *GetDemand()* in C or *getDemand()* in Java
  - Add a new function *GetDeliveryLag()* in C or *getDeliveryLag()* in Java
    - Use *Uniform*$(0, 1)$ random variate delivery lag.
- Calculate statistics: average order, average demand, relative frequency of setups, time-averaged holding level, and time-averaged shortage level, and dependent cost (i.e., the sum of the average setup, holding, and shortage costs)

Submit the program, the simulation result, and the graph as in Example 3.3.4 (e.g., Excel Workbook) in Blackboard

In-Class Exercise L7-2: Recap

Complete the consistency checks as outlined in slides 22 and 23 using the instructor's sample solution.

- Compute statistics when the delivery lag is generated using random variates $Uniform(0, 0.00)$, $Uniform(0, 0.01)$, $Uniform(0, 0.02)$, ... and $Uniform(0, 0.10)$.
- Perform consistency checks:
  - Special value: we expect that the statistics should be the same as those obtained from *sis2* when the delivery lag is 0
  - Small-perturbation: when the delivery lag is small, we expect that the statistics should be different from those obtained from *sis2*, but the different should be small.
  - Intuition: if there is a delivery lag, we expect to observe that average holding should decrease and average shortage should increase when compared to the lag-free system; average order, average demand, and order frequency are independent of delivery lags.
- Use a well-designed graph to show the results of the consistency checks. Submit the graph in Blackboard.

Single-Server Machine Shop

Let us consider a machine shop.

- It has a finite number of identical machines.
- Each machine operates continuously until failure.
- As machines fail, they are repaired, in the order in which they fail, by a service technician.
- As soon as a failed machine is repaired, it is put back into operation.

The machines when they are operate produce an income.

- e.g., a machine operates 8 hours per day, 250 days a year, and produces a net income of $20 per hour of operation.

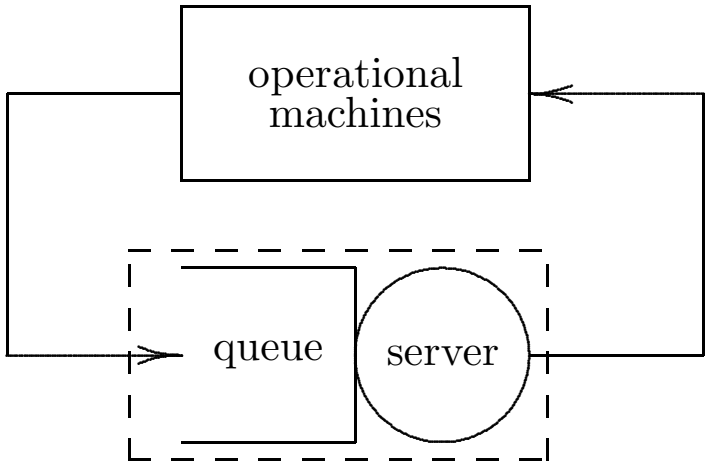
The service technician takes time to repair machine which leads to a cost.

- Hiring a technican costs money.
- However, more technicans would allow machines to be repaired more quickly and put back to operations, which leads to more income.

The *objective* is how one may maximize the profit by hiring the right number of technicans.

Single-Server Machine Shop: Specification

- The machine shop model is *closed* because there are a finite number of machines in the system.



- Assume repair times are *Uniform*$(1.0, 2.0)$ random variates
  - To reduce the average pair times, one may hire more technicians.
- There are M machines that fail after an *Exponential*(100.0) random variate

Single-Server Machine Shop: Computational Model

Single-Server Machine Shop

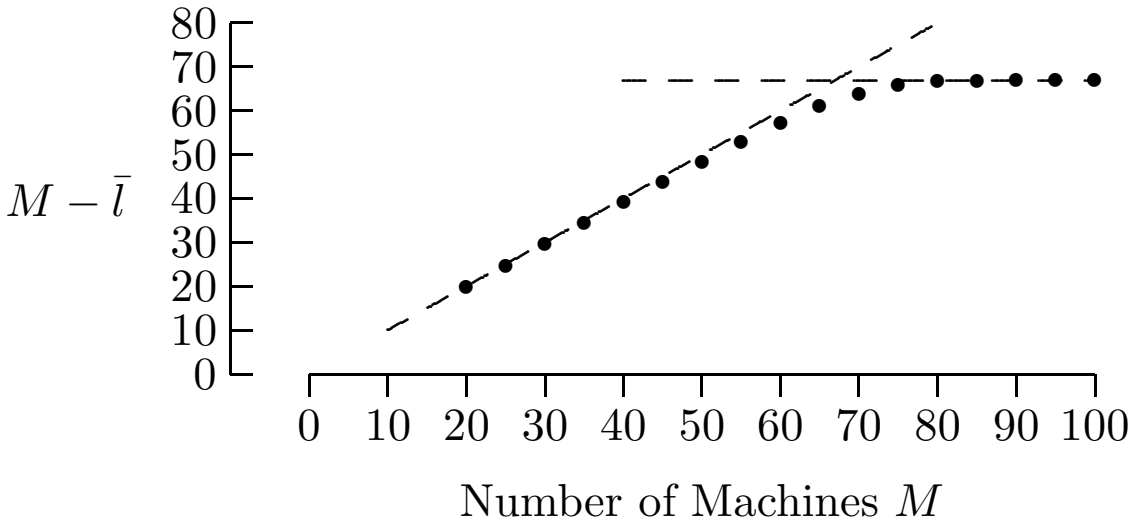
$C_0 = 0.0$;
$i = 0$;
for each machine i in $\{m_1, m_2, \ldots m_M\}$
  initialize f[i];
while (more failure to process) {
  i ++;
  $(a_i, m) = NextFailure()$;
  if $(a_i < c_{i-1})$
    $d_i = c_{i-1} - a_i$;
  else
    $d_i = 0.0$;
  $s_i = GetSerivce()$;
  $c_i = a_i + d_i + s_i$;
  $f[m] = c_i + GetFailure()$;
}
$n = i$;
return $d_1, d_2, \ldots, d_n$;

Single-Server Machine Shop: Computational Model

- Program *ssms* simulates a single-server machine shop
- The library *rngs* is used to uncouple the random processes
- The failure process is defined by the array *failures*
  - A $\mathcal{O}(M)$ search is used to find the next failure
  - Alternate data structures can be used to increase computational efficiency

Example 3.3.5

- The time-averaged number of working machines is $M - \bar{l}$



- For small values of M the time-averaged number of operational machines is essentially M
- For large values of M this value is essentially constant at approximately 67

In-Class Exercise L7-3

Complete the following tasks using *ssms*

- Relative to Example 3.3.5 in slide 31, construct a figure illustrating how $\overline{x}$ (utilization) depends on M.
- If you extropolate linearly from small values of M, at what value of M will saturation ($\overline{x} = 1$) occur?
- Can you provide an empirical argument or equation to justify this value?

Submit the answers to Blackboard.

Summary

Three examples:

- SSQ with immediate feedback
- SIS with delivery lag
- Single-Server Machine shop