CISC 7332X T6
C05b: Some Foundation of Data Communication

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Outline

- Concept of Fourier analysis
- Bandwidth and bandwidth-limited signals
- Maximum data rate of a noiseless channel
- Maximum data rate of a noisy channel
- Wave length and propagation speed

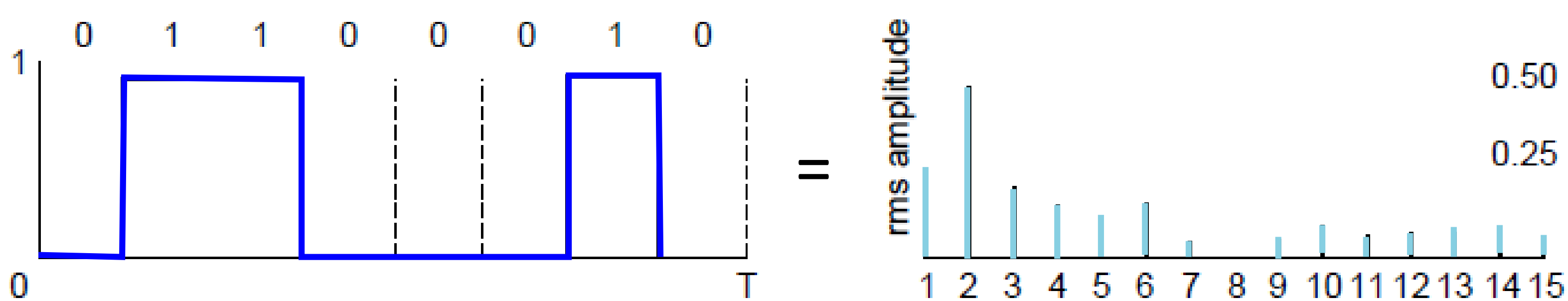
Theoretical Basis for Data Communication

- Communication rates have fundamental limits
  - Fourier analysis
  - Bandwidth-limited signals
  - Maximum data rate of a channel

Fourier Analysis

- A time-varying signal can be equivalently represented as a series of frequency components (harmonics)

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n \sin(2\pi n f t) + \sum_{n=1}^{\infty} b_n \cos(2\pi n f t)$$

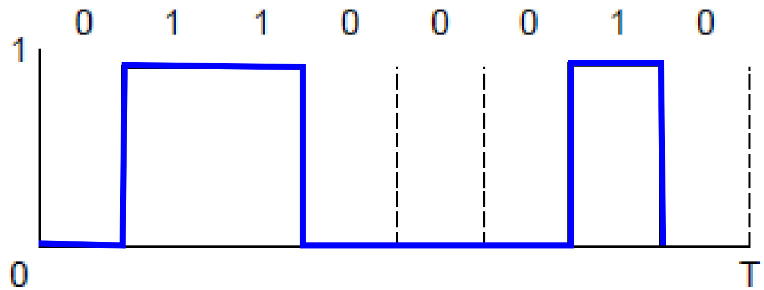


Signal over time ⟶

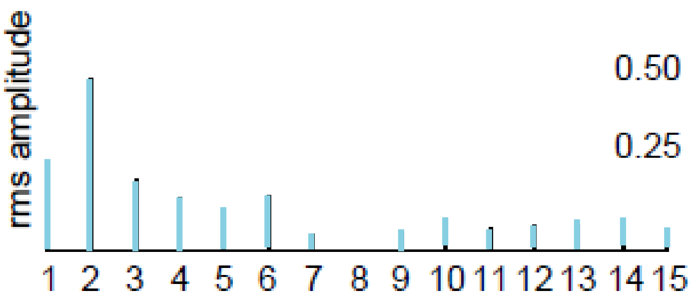
a, b weights of harmonics

Bandwidth-Limited Signals

- Bandwidth
  - The width of the frequency range transmitted without being strongly attenuated is called the bandwidth
- Having less bandwidth (harmonics) degrades the signal

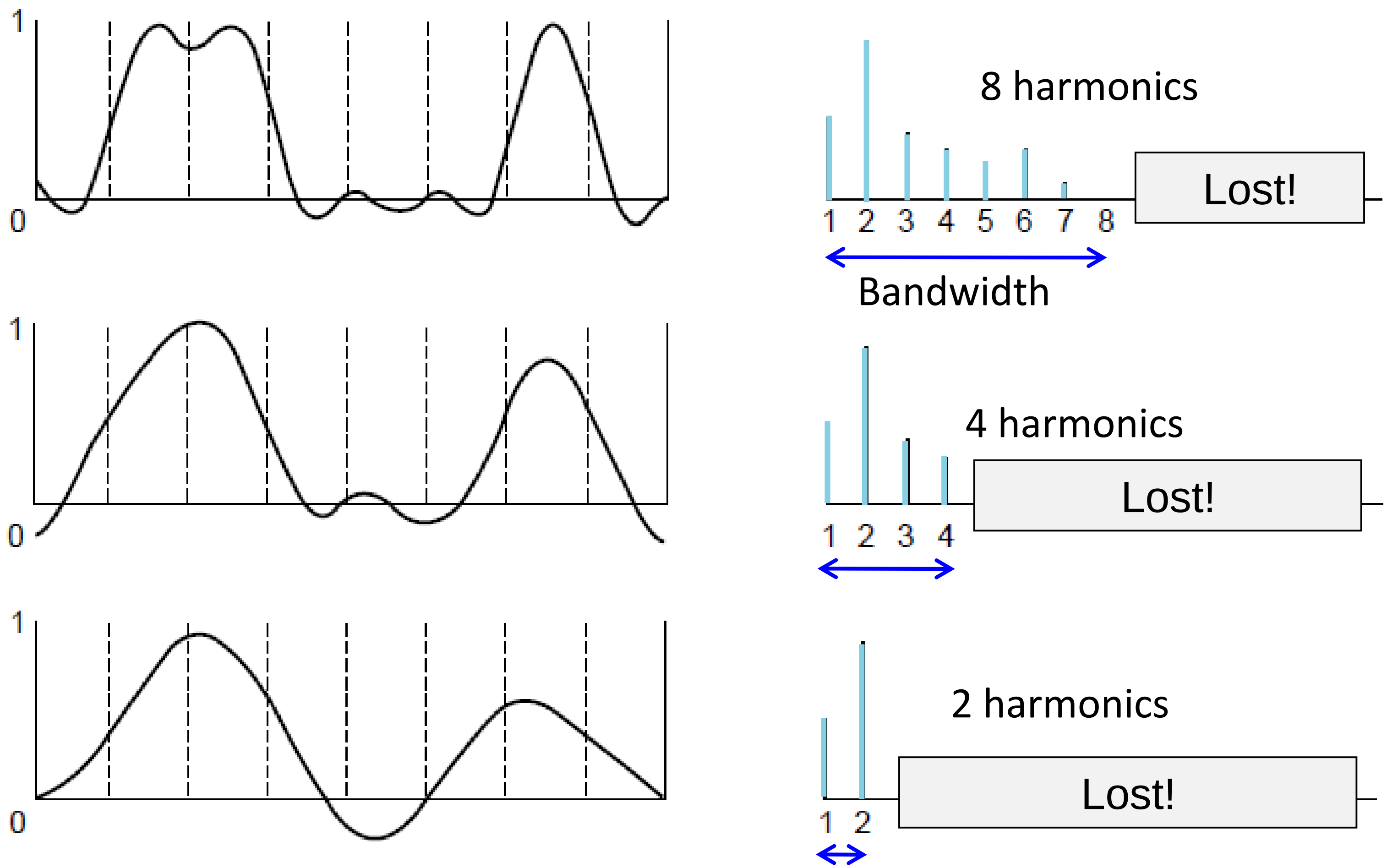


Signal over time →



a, b weights of harmonics

Bandwidth-Limited Signals



Maximum Data Rate of a Channel

- Noiseless channel
- Noisy channel

Maximum Data Rate of Noiseless Channel

- Nyquist's theorem relates the data rate to the bandwidth (B) and number of signal levels (V):

$$\text{Max. data rate} = 2B \log_2 V \text{ bits/sec}$$

Example: A Noiseless Channel

- How much is the maximum data rate when transmitting signals of 2-levels over a noiseless 3-KHz channel?
  - which means
    - B = 3 kHz = 3000 Hz
    - V = 2

Max. data rate

$$= 2B \log_2 V \text{ bits/sec}$$

$$= 2 \times 3000 \times \log_2 2$$

$$= 6000 \text{ bits / sec}$$

Maximum Data Rate of Noisy Channel

- Shannon's theorem relates the data rate to the bandwidth (B) and signal strength (S) relative to the noise (N):

$$\text{Max. data rate} = \underbrace{B}_{\text{How fast signal can change}} \log_2 \underbrace{(1 + S/N)}_{\text{How many levels can be seen}} \text{ bits/sec}$$

Signal-to-Noise Ratio

- S/N: the signal-to-noise ratio
- Often measured in log-scale, i.e., in decibels (dB)
  - 1 dB = 1 decibel = 1 deci-Bel = 1/10 Bels

$$\text{SNR in dB} = \underbrace{10}_{\text{deci}} \underbrace{\log_{10} S/N}_{\text{Bel}} \text{ dB}$$

Example: an ADSL Channel

- Asymmetric Digital Subscriber Line (ADSL) provides Internet access over ordinary telephone lines. Consider an ADSL channel with the bandwidth of 1 Mhz. The SNR depends strongly on the distance of the home from the telephone exchange, and an SNR of ~40dB for short lines of 1 or 2 km is very good. How much is the maximum date rate?

Example: an ADSL Channel

The ADSL channel

- $B = 1$ Mhz $= 10^6$ Hz
- SNR = 40 dB

Estimate S/N

- $10 \log_{10} (S/N) = 40$
- $S/N = 10^4 = 16$

Max. data rate

$= B \log_2(1 + S/N)$ bits/sec

$= 10^6 \times \log_2(1+10^4)$

$\approx 10^6 \times 13.29$ bits/sec = 13.29 Mbps

In-Class Exercise C05b-1

Television channels are 6 MHz wide. Answer the questions

1) How many bits/sec can be sent if 4-level digital signals are used? Assume a noiseless channel

2) Assume that it is a noisy channel with a SNR 30 dB. How many bits/sec can be sent?

In-Class Exercise C05b-2

Television channels are 6 MHz wide. Answer the questions

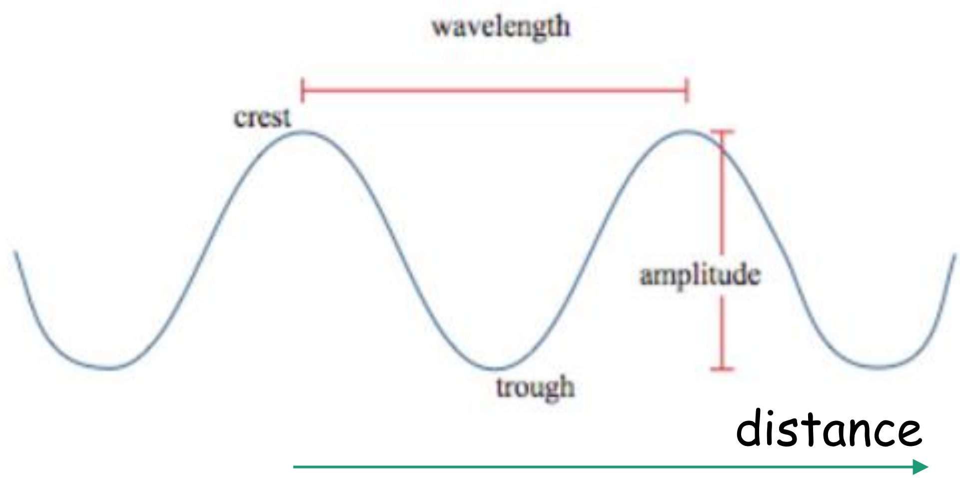
1) Assume a noiseless channel. What is the minimum levels of the digital signals is necessary to reach data rate 5 Mbps?

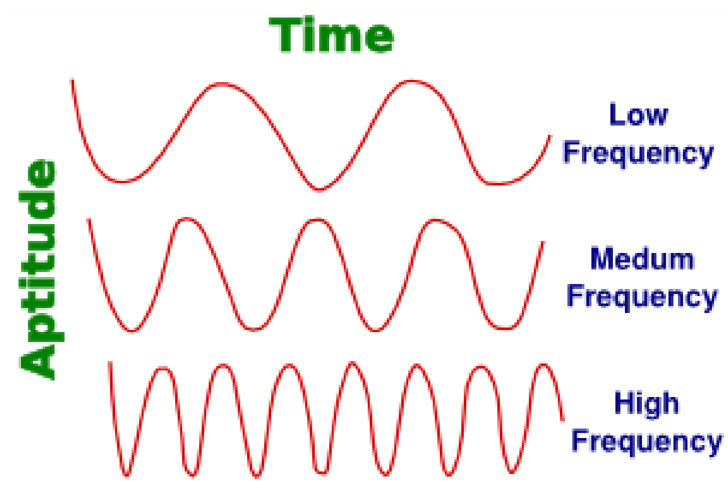
2) Assume that it is a noisy channel and you wish to reach a maximum date rate of 5 Mbps. What signal-to-noise ratio is needed? In dB?

Signal and Wave

- Wave length (λ)
- Frequency (f)
- Wave speed (v)

$$v = \lambda f$$





Example: Light in Vacuum

- Consider a visible light of 500 THz traveling in vacuum. How much is the wave length?
- Wave speed
  - v = speed of light in vacuum = $c \approx 3 \times 10^8$ meter/sec
  - $f = 500$ THz $= 500 \bullet 10^{12}$ Hz $= 500 \bullet 10^{12}\ \text{sec}^{-1}$
  - $\lambda = v / f = 3 \times 10^8 / (500 \bullet 10^{12})$

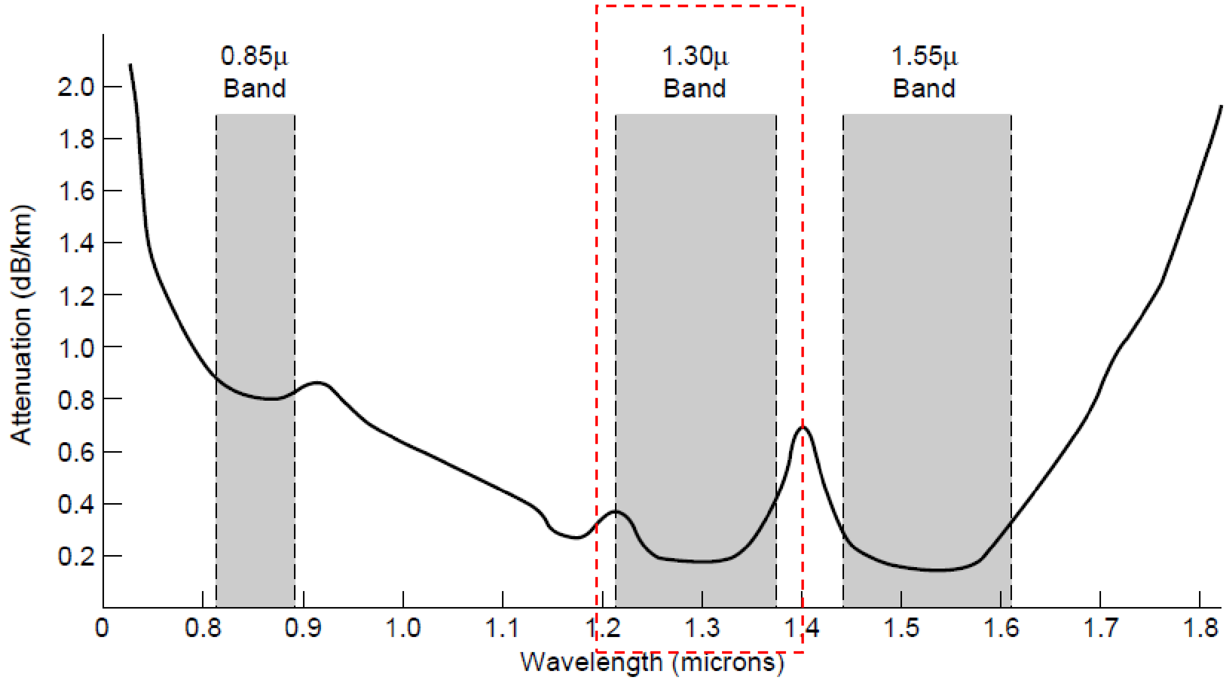
    $= 6 \bullet 10^{-7}$ meters

    $= 600 \bullet 10^{-9}$ meters

    = 600 nanometers

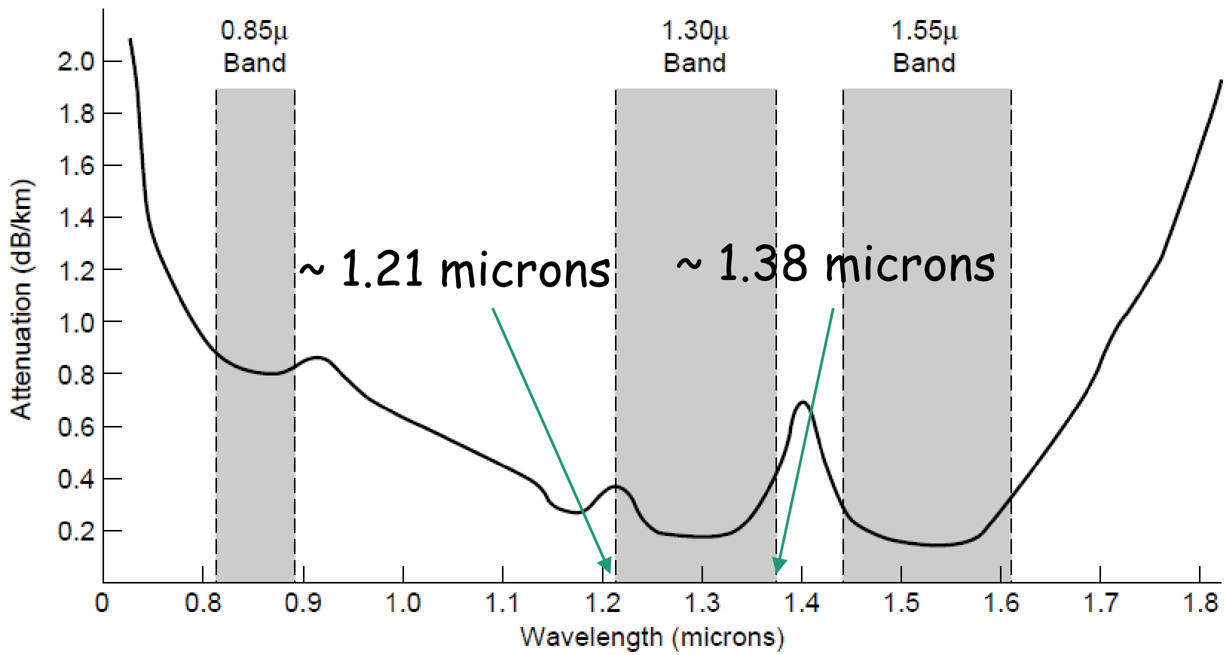
Example: Bandwidth of a Fiber

- Consider the 1.30-micron (micro-meter) band in the figure. With a reasonable signal-to-noise ratio of 10 dB, how much is the maximum data rate?



Example: Bandwidth of a Fiber

- Read from the graph, estimate the bandwidth



Example: Bandwidth of a Fiber

Approximately,

$v = c \approx 3 \times 10^8$ meter/sec

$f_{high} = v / \lambda_{low} \approx 3 \times 10^8 / (1.21 \times 10^{-6}) \approx 2.48 \times 10^{14}$ Hz

$f_{low} = v / \lambda_{high} \approx 3 \times 10^8 / (1.38 \times 10^{-6}) \approx 2.17 \times 10^{14}$ Hz

$B \approx (2.48 - 2.17) \times 10^{14} = 0.31 \times 10^{14}$ Hz

Since $10 \log_{10} S/N = 10$ dB,

$S/N = 10$

Then

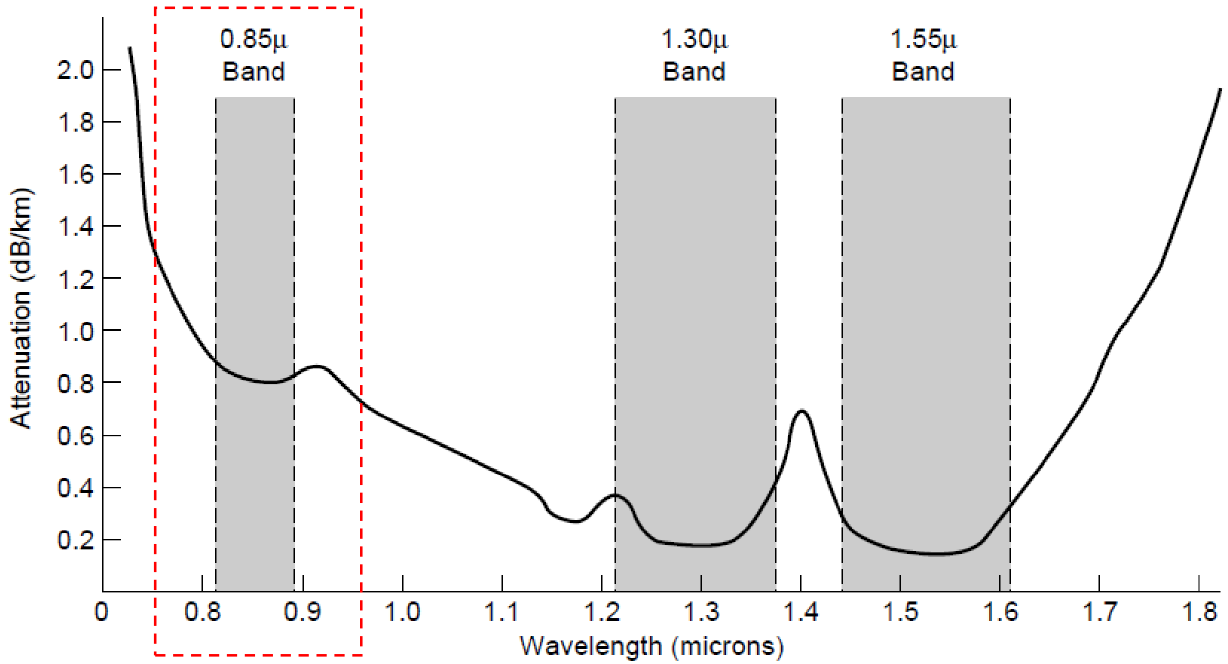
Max. data rate $= B \log_2(1 + S/N)$

$\approx 0.31 \times 10^{14} \times \log_2(1 + 10) = 0.107 \times 10^{15}$ bits/sec

$= 107 \times 10^{12}$ bits/sec $= 107$ Tbps

In-Class Exercise C05b-3

- Consider the 0.85-micron (micro-meter) band in the figure. With a reasonable signal-to-noise ratio of 10 dB, how much is the maximum data rate?



Questions?

- Concept of Fourier analysis
- Bandwidth and bandwidth-limited signals
- Maximum data rate of a noiseless channel
- Maximum data rate of a noisy channel
- Relationship among wavelength, frequency, and wave speed
- Exercises and assignments?