CISC 3115 EWQ6
Recursion and Recursive Math Functions

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Outline

- Problem Solving using Recursion
- Recursive math functions
- Design solutions to recursive math functions using recursion
  - Define mathematical recursive function with base case
  - Design Java methods

Problem Solving using Recursion

- A divide-and-conquer problem solving approach where a problem can be divided into the same problems of smaller size
- Examples
  - Mathematical recursive functions
  - Sorting, searching
  - …

Mathematical Recursive Functions

- Such functions take their name from the process of recursion by which the value of a function is defined by the application of the same function applied to smaller arguments.
- Examples
  - Function to compute factorials
  - Function to compute Fibonacci numbers

Factorial

- Factorial of n is defined as
  - $f(n) = n! = n\ (n-1)\ (n-2)\ \ldots\ 1$
- whose recursive function can be
  - $f(n) = n\ f(n-1)$
- with the base case
  - $f(0) = 1$

The same problem of smaller size

The same function applied to smaller arguments

Computer Factorial

- Recursive function to compute factorial

$$f(n) = \begin{cases} nf(n-1) & if\ n > 0 \\ 1 & if\ n = 0 \end{cases}$$

- Example
  - f(4) = 4 f(3) = 4 3 f(2) = 4 3 2 f(1) = 4 3 2 1 f(0) = 4 3 2 1 1 = 24

Base Case

- Base case is important
- Otherwise, where do we stop (without the base case)? e.g., consider
  - f(3) = 3 f(2) = 3 2 f(1) = 3 2 1 f(0) = 3 21 0 f(-1) = 3 2 1 0 -1 f(-2) …
- The base case makes sure that we stop the recursive process somewhere.

Design Factorial Recursive Method

- Design: int factorial(int n)
- Observe:
  - Recursive function: f(n) = n * f(n-1) when n > 0
  - Bae case: f(0) = 1
- Design method factorial(n: int):
  - f(n) = n * f(n-1): when computing f(n), we invoke factorial(n) where we compute it by n * factorial(n-1), i.e., we invoke the same factorial method recursively.
  - f(0) = 1: we stop invoking the factorial method when n is 0.

Fibonacci Number

- Mathematical recursive function to compute Fibonacci numbers

$$f(n) = \begin{cases} f(n-1) + f(n-2) & if\ n > 1 \\ 1 & if\ n = 1 \\ 0 & if\ n = 0 \end{cases}$$

- What is the base case?

Design Fibonacci Recursive Method

- Design: int fibonacci(int n)
- fibonacci(n) is computed as
  - fibonacci(n-1)+fibonacci(n-2) when n>1 based on recursive function
  - $f(n) = f(n-1) + f(n-2)$ when $n>1$
- fibonacci(0) should return 0 and fibonacci(1) should return 1 according to the base case
  - $f(0) = 0$
  - $f(1) = 1$

Recursive Calls and Call Stack

- factorial(4) = 4 * factorial(3)

  = 4 * (3 * factorial(2))

  = 4 * (3 * (2 * factorial(1)))

  = 4 * (3 * (2 * (1 * factorial(0))))

  = 4 * (3 * (2 * (1 * 1))))

  = 4 * (3 * (2 * 1))

  = 4 * (3 * 2)

  = 4 * (6)

  = 24

- Observe the animation from the publisher and the author of the textbook (included below)

animation

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

factorial(4)

animation

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)

animation

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)

= 4 * 3 * factorial(2)

animation

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)

= 4 * 3 * factorial(2)

= 4 * 3 * (2 * factorial(1))

animation

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)

= 4 * 3 * factorial(2)

= 4 * 3 * (2 * factorial(1))

= 4 * 3 * (2 * (1 * factorial(0)))

animation

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)

= 4 * 3 * factorial(2)

= 4 * 3 * (2 * factorial(1))

= 4 * 3 * (2 * (1 * factorial(0)))

= 4 * 3 * (2 * (1 * 1)))

animation

Computing Factorial

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= 4 * 3 * (2 * (1 * 1)))

= 4 * 3 * (2 * 1)

animation

Computing Factorial

factorial(0) = 1;

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= 4 * 3 * (2 * (1 * factorial(0)))

= 4 * 3 * (2 * (1 * 1)))

= 4 * 3 * (2 * 1)

= 4 * 3 * 2

animation

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

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= 4 * (3 * factorial(2))

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= 4 * (3 * (2 * (1 * 1))))

= 4 * (3 * (2 * 1))

= 4 * (3 * 2)

= 4 * (6)

animation

Computing Factorial

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= 4 * (3 * (2 * factorial(1)))

= 4 * (3 * (2 * (1 * factorial(0))))

= 4 * (3 * (2 * (1 * 1))))

= 4 * (3 * (2 * 1))

= 4 * (3 * 2)

= 4 * (6)

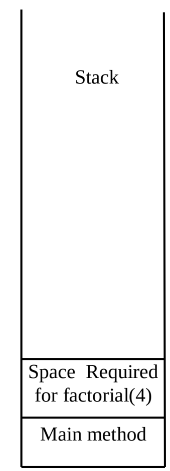
= 24

animation

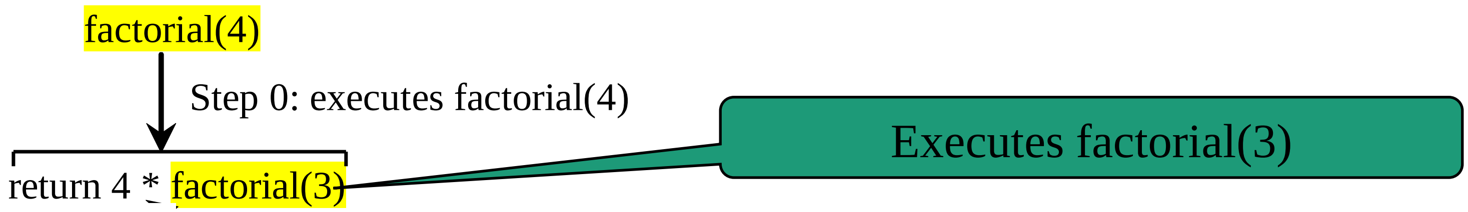
Trace Recursive factorial

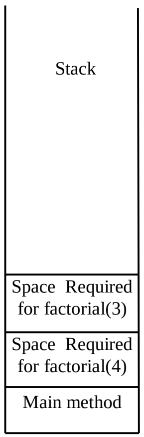
Executes factorial(4)

factorial(4)



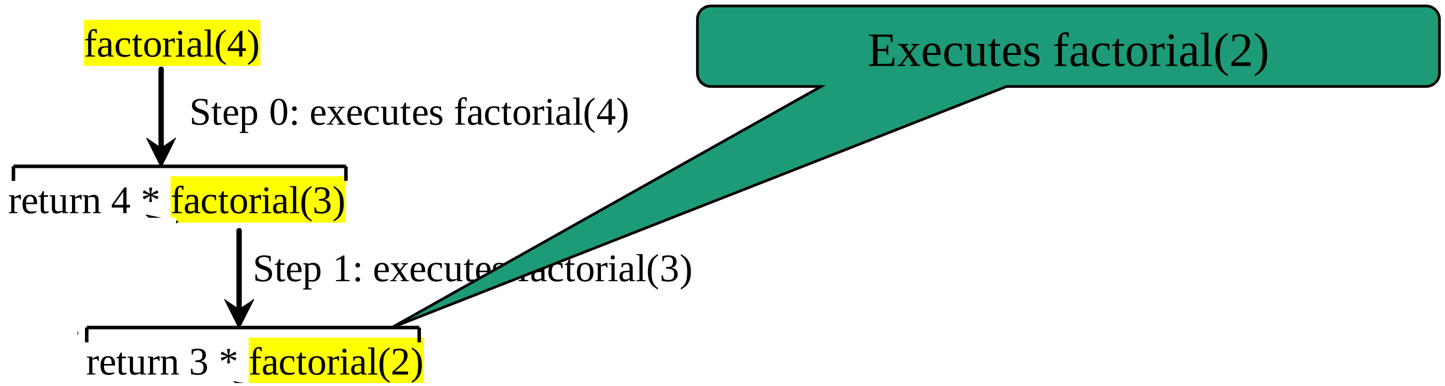
Trace Recursive factorial

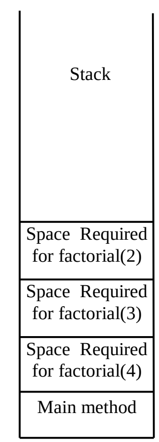




animation

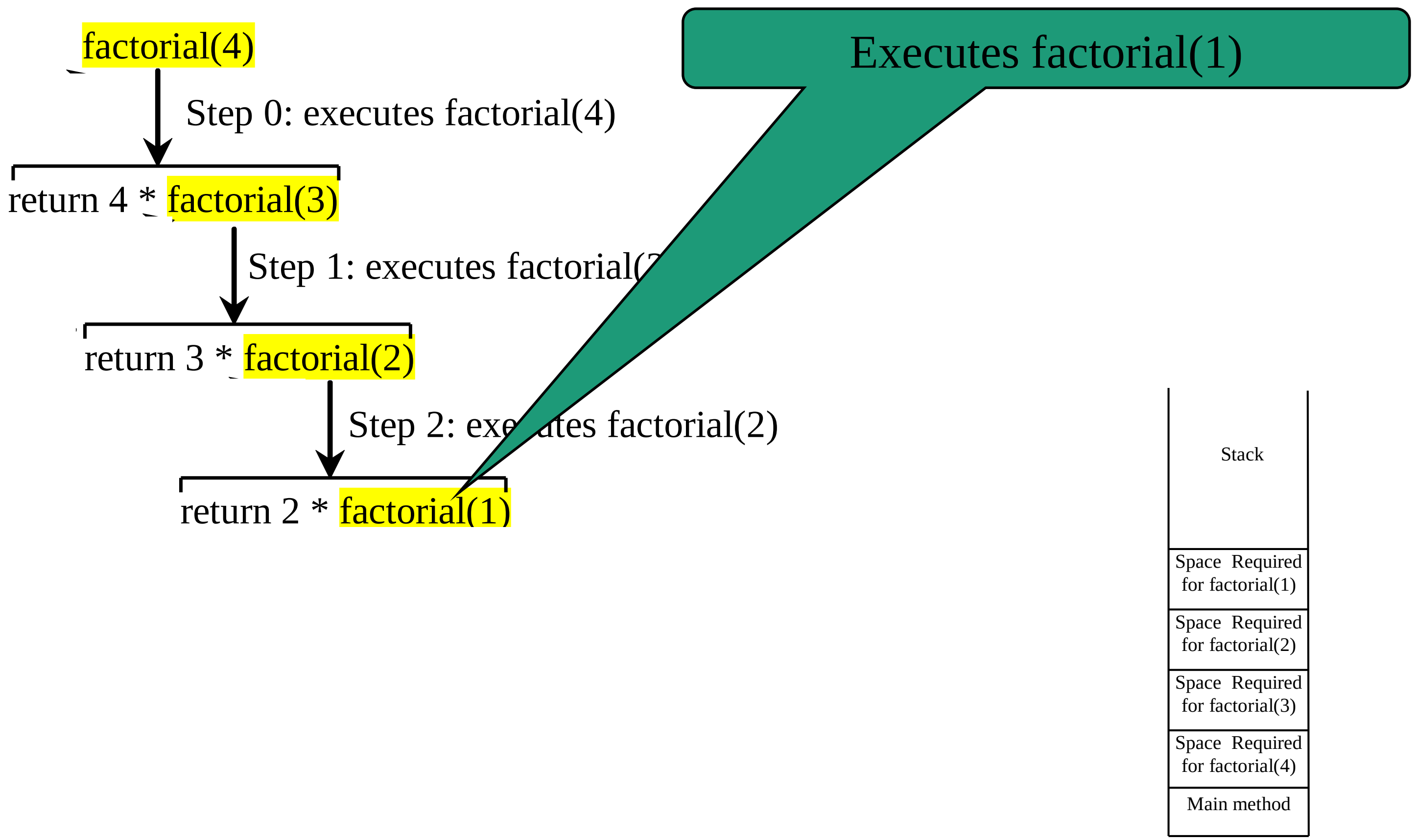
Trace Recursive factorial





animation

Trace Recursive factorial



animation

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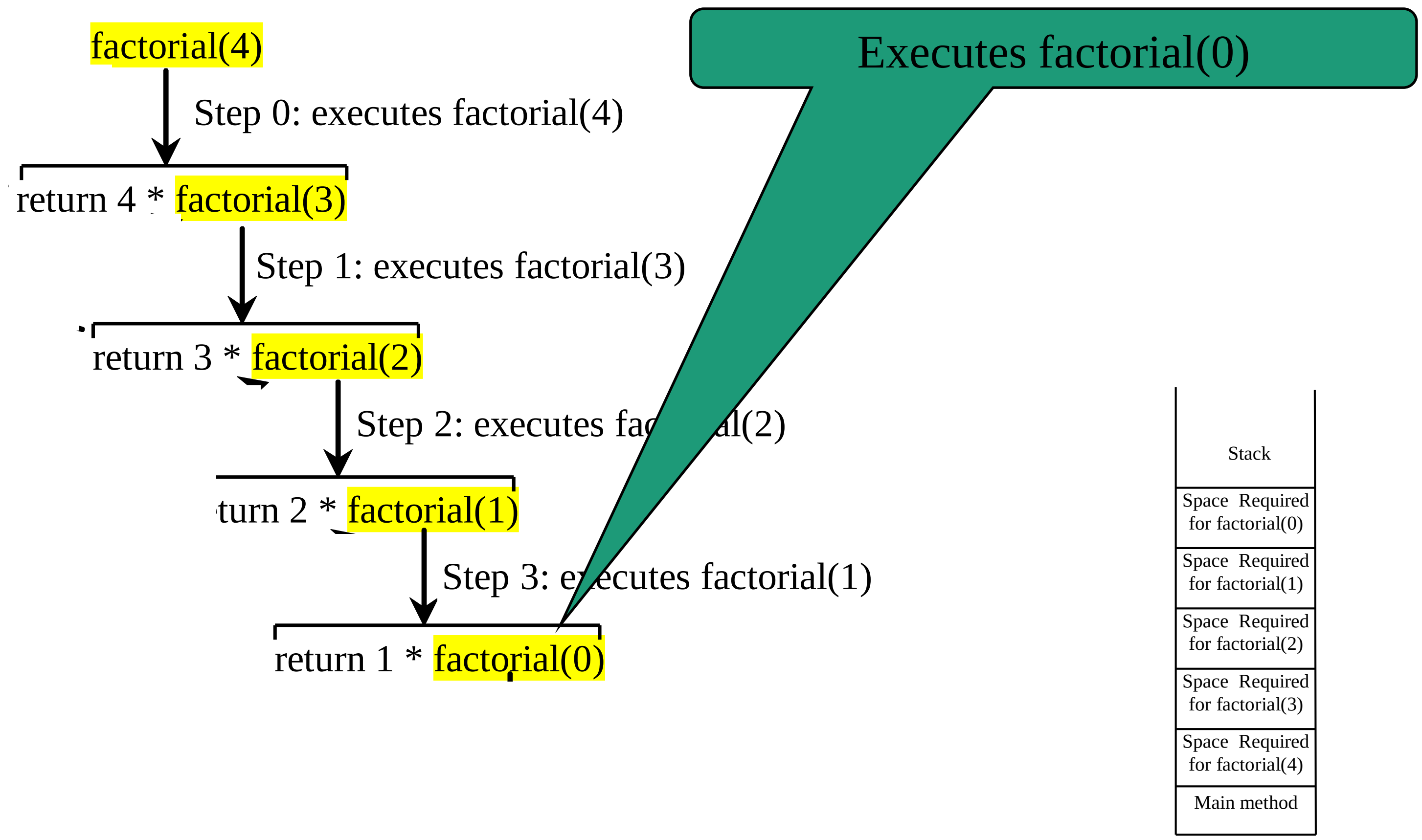
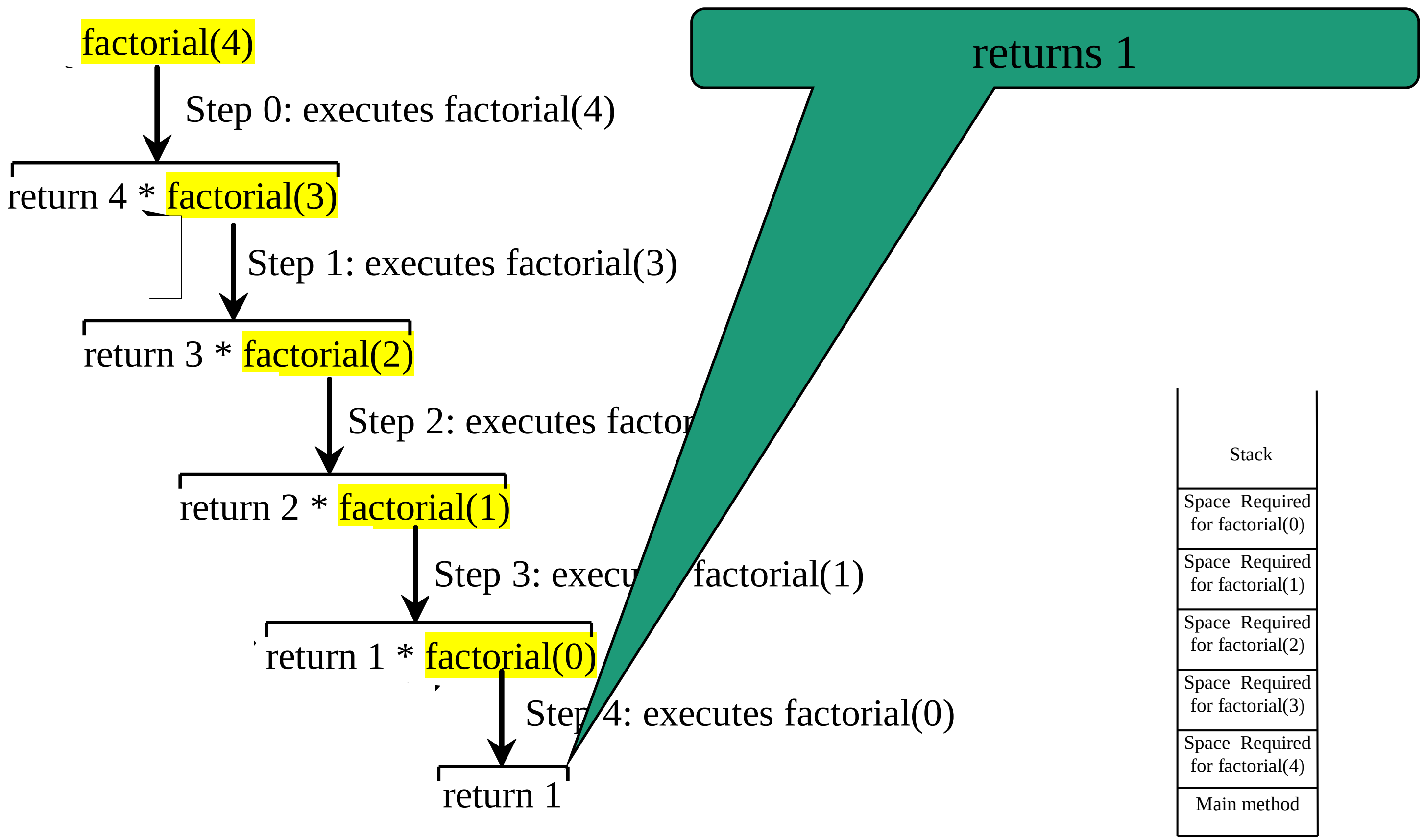


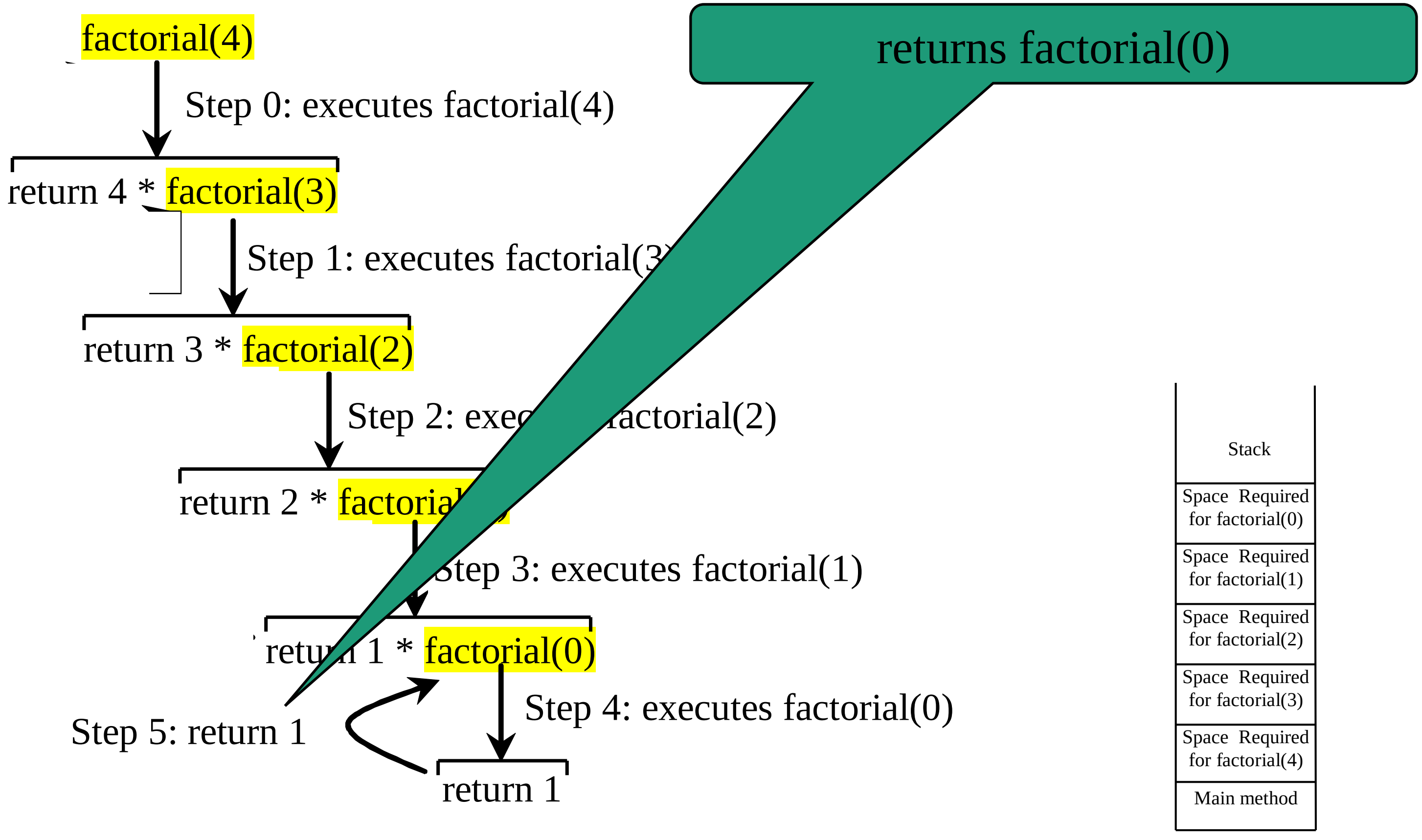
animation

Trace Recursive factorial



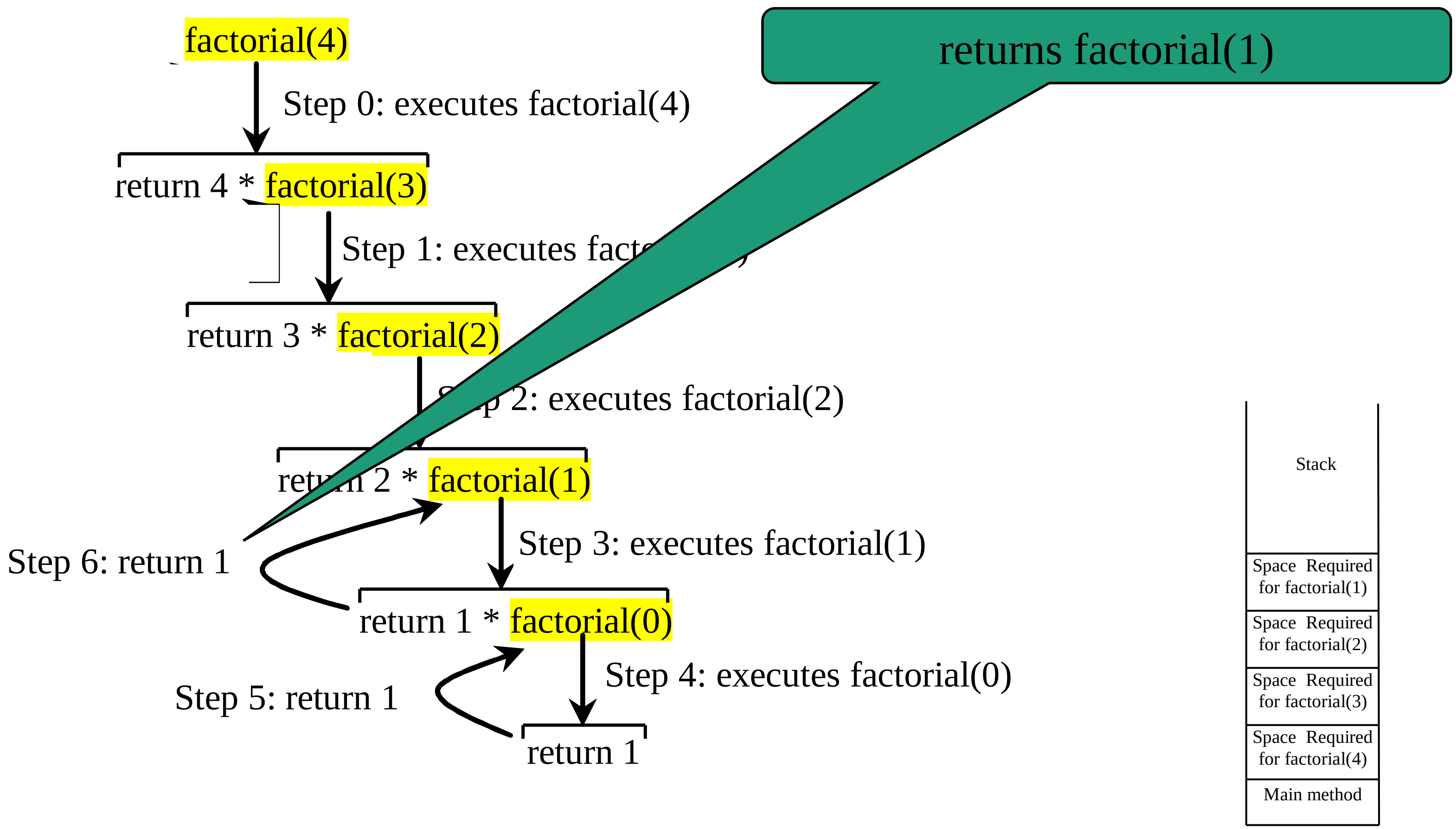
animation

Trace Recursive factorial



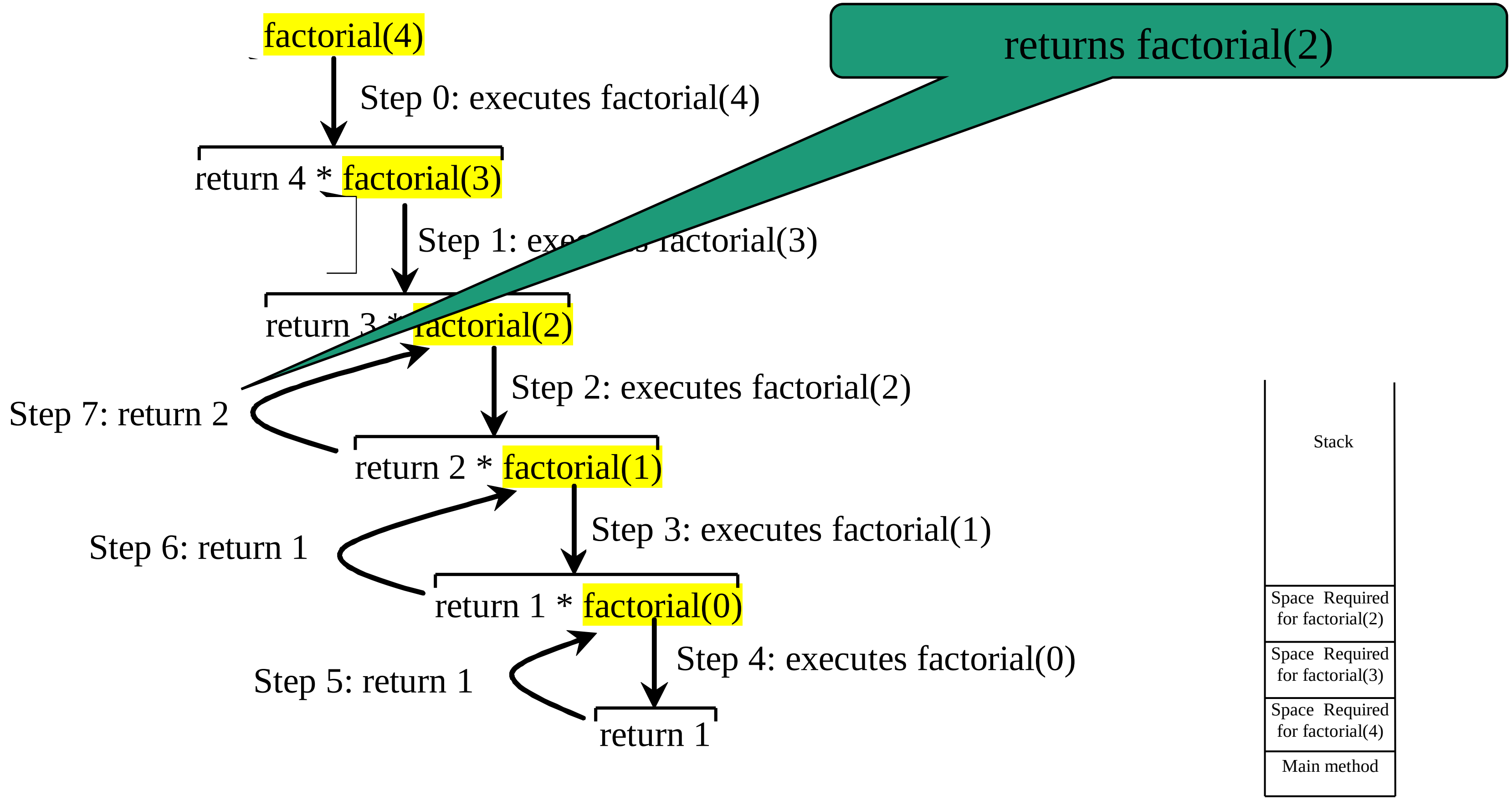
animation

Trace Recursive factorial



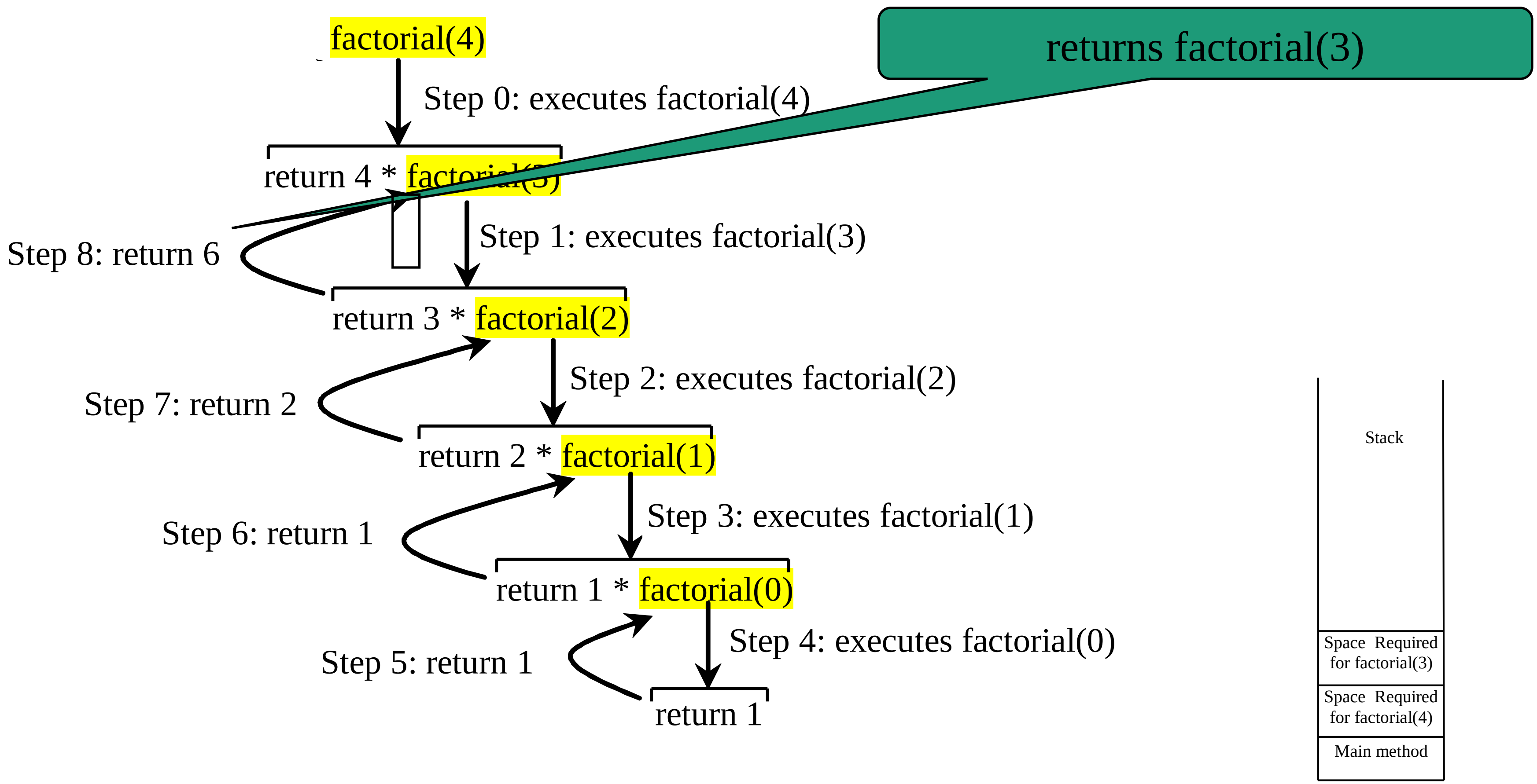
animation

Trace Recursive factorial

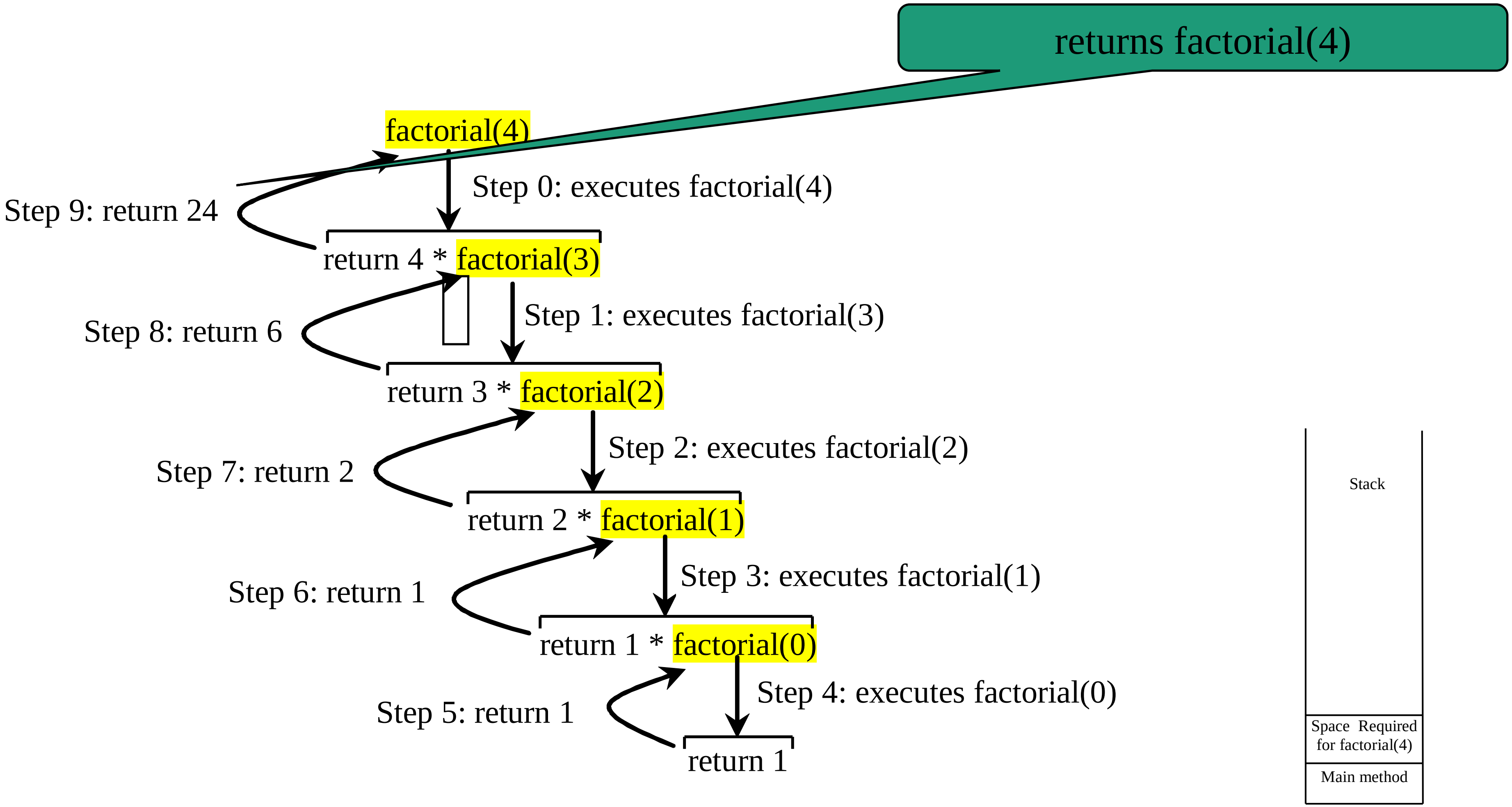


animation

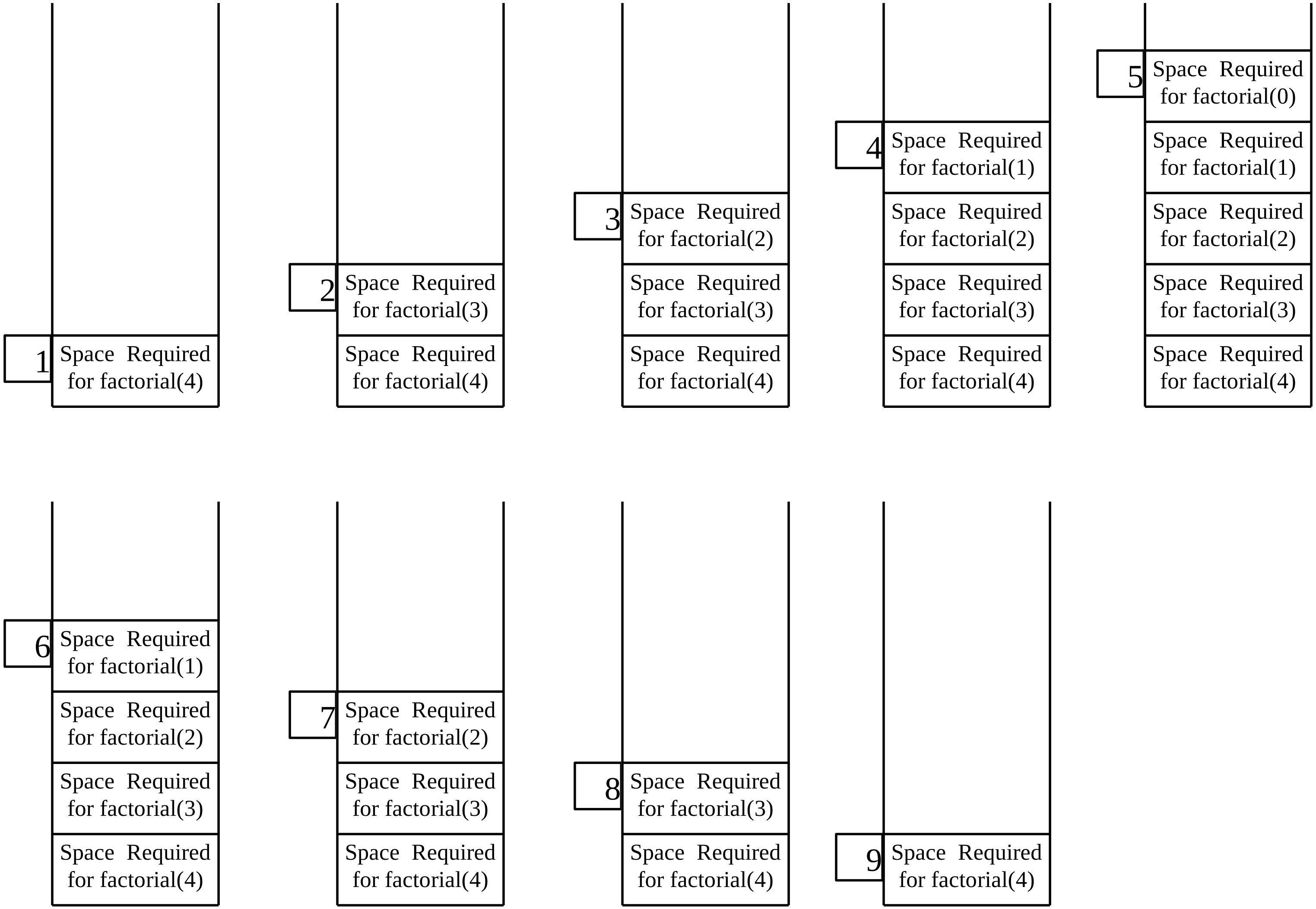
Trace Recursive factorial



Trace Recursive factorial



factorial(4) Stack Trace



Stack Overflow Error

- Neglecting or mishandling the base case will lead to a Stack Overflow error, for which, Java throws a StackOverflowError

```
$ java Factorial
Exception in thread "main" java.lang.StackOverflowError
        at Factorial.factorial(Factorial.java:3)
        at Factorial.factorial(Factorial.java:3)
        at Factorial.factorial(Factorial.java:3)
        at Factorial.factorial(Factorial.java:3)
...
```

Characteristics of Recursion

- All recursive methods have the following characteristics:
  - One or more base cases (the simplest case) are used to stop recursion.
  - Every recursive call reduces the original problem, bringing it increasingly closer to a base case until it becomes that case.

Recursion as Problem Solving Strategy

- Break the problem into subproblems such that one or more subproblems resembles the original problem
  - These subproblems resembling the original problem is almost the same as the original problem in nature with a smaller size.
- Apply the same approach to solve the subproblem recursively to reach the base case

Questions?

- Concept of recursion
- Problem solving using recursion
  - Mathematical recursive functions
  - Base case
  - Call stack and stack trace
  - StackOverflowError